

Unit 9 Variation Worksheet

① $(x^2-1)^2 + 2(x^2-1) - 3 = 0$

let $y = x^2 - 1$

$y^2 + 2y - 3 = 0$

$(y+3)(y-1) = 0$

$y = -3$

$y = 1$

$x^2 - 1 = -3$

$x^2 - 1 = 1$

$x^2 = -2$

$x^2 = 2$

$x = \pm i\sqrt{2}$

$x = \pm \sqrt{2}$

$\{\pm i\sqrt{2}, \pm \sqrt{2}\}$

② $x^2 + 3x + \sqrt{x^2 + 3x - 2} = 22$

$x^2 + 3x - 2 + \sqrt{x^2 + 3x - 2} - 20 = 0$

let $y = \sqrt{x^2 + 3x - 2}$

$y^2 = x^2 + 3x - 2$

$y^2 + y - 20 = 0$

$(y+5)(y-4) = 0$

$y = -5$ $y = 4$

$\sqrt{x^2 + 3x - 2} = -5$

$\sqrt{x^2 + 3x - 2} = 4$

$x^2 + 3x - 2 = 16$

$x^2 + 3x - 18 = 0$

$(x+6)(x-3) = 0$

$x+6=0$

$x-3=0$

$x = -6$

$x = 3$

$\{-6, 3\}$

③ $\frac{3}{4}x^{-\frac{2}{3}}(5x^{\frac{1}{3}} - 2) = 0$

let $y = x^{\frac{1}{3}}$

$\frac{3}{4}y^{-2}(5y - 2) = 0$

$\frac{3}{4}y^{-2} = 0$ $5y - 2 = 0$

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$y = \frac{2}{5}$

$x^{\frac{1}{3}} = \frac{2}{5}$

$\left\{\frac{8}{125}\right\}$ $x = \left(\frac{2}{5}\right)^3$

$x = \frac{8}{125}$

④ $\frac{x}{3}(x^2+4)^{-\frac{1}{2}} - \frac{1}{5} = 0$

$\frac{x}{3(x^2+4)^{\frac{1}{2}}} = \frac{1}{5}$

$[5x = 3(x^2+4)^{\frac{1}{2}}]^2$

$25x^2 = 9(x^2+4)$

$25x^2 = 9x^2 + 36$

$16x^2 = 36$

$x^2 = \frac{9}{4}$ $-\frac{3}{2}$ (extraneous)

$x = \pm \frac{3}{2}$ $\left\{\frac{3}{2}\right\}$

⑤ $-\frac{1}{4(\sqrt{x})^3} + \frac{2}{2(\sqrt{x})^5} = 0$

$\frac{2}{2(\sqrt{x})^5} = \frac{1}{4(\sqrt{x})^3}$

$4(\sqrt{x})^3 = (\sqrt{x})^5$

$4 = (\sqrt{x})^2$

$4 = x$

⑥ $2\sqrt[3]{1-x} + 3\sqrt[3]{1-x} = 2$

let $y = \sqrt[3]{1-x}$

$\therefore y^3 = 1-x$

$2y^2 + 3y = 2$

$2y^2 + 3y - 2 = 0$

$(2y-1)(y+2) = 0$

$y = \frac{1}{2}$ $y = -2$

$\sqrt[3]{1-x} = \frac{1}{2}$ $\sqrt[3]{1-x} = -2$

$1-x = \frac{1}{64}$

$\frac{63}{64} = x$

$\left\{\frac{63}{64}\right\}$

⑦ $\frac{x^2+2x}{3} + \frac{3}{x^2+2x} = \frac{26}{5}$

let $y = x^2+2x$

$\frac{y}{3} + \frac{3}{y} = \frac{26}{5}$

$\frac{y^2+9}{3y} = \frac{26}{5}$

$5y^2 + 45 = 78y$

$5y^2 - 78y + 45 = 0$

$(5y-3)(y-15) = 0$

$y = \frac{3}{5}$ $y = 15$

$x^2+2x = \frac{3}{5}$

$5x^2+10x-3=0$

$a=5$ $b=10$ $c=-3$

$x = \frac{-10 \pm \sqrt{100 - 4(5)(-3)}}{10}$

$x = \frac{-10 \pm \sqrt{160}}{10}$

$x = \frac{-10 \pm 4\sqrt{10}}{10} = \frac{-5 \pm 2\sqrt{10}}{5}$

$y = 15$

$x^2+2x = 15$

$x^2+2x-15=0$

$(x+5)(x-3)=0$

$x = -5$ $x = 3$

$\left\{-5, 3, \frac{-5 \pm 2\sqrt{10}}{5}\right\}$

Variation Worksheet - continued

Unit 9

$$\textcircled{8} x^6 + 7x^3 - 8 = 0$$

$$(x^3 - 1)(x^3 + 8) = 0$$

$$(x-1)(x^2+x+1)(x+2)(x^2-2x+4) = 0$$

$$x-1=0 \quad x^2+x+1=0 \quad x+2=0 \quad x^2-2x+4=0$$

$$x=1 \quad a=1 \quad b=1 \quad c=1$$

$$x=-2 \quad a=1 \quad b=-2 \quad c=4$$

$$x = \frac{-1 \pm \sqrt{1-4(1)(1)}}{2}$$

$$x = \frac{2 \pm \sqrt{4-4(1)(4)}}{2}$$

$$x = \frac{-1 \pm \sqrt{-3}}{2}$$

$$x = \frac{2 \pm \sqrt{-12}}{2}$$

$$x = \frac{-1 \pm i\sqrt{3}}{2}$$

$$x = \frac{2 \pm 2i\sqrt{3}}{2}$$

$$x = 1 \pm i\sqrt{3}$$

$$\left\{ 1, -2, \frac{-1 \pm i\sqrt{3}}{2}, 1 \pm i\sqrt{3} \right\}$$

$$\textcircled{10} L = \frac{Kbd^2}{2}$$

$$L = \frac{375bd^2}{2L}$$

$$750 = \frac{K(2)(4)^2}{8}$$

$$L = \frac{375(2)(6)^2}{2(8)}$$

$$750 = \frac{32K}{8}$$

$$L = 1687.5 \text{ lbs}$$

$$750 = 4K$$

$$K = \frac{750}{4} = \frac{375}{2}$$

$$\textcircled{9} F = \frac{K m_1 m_2}{d^2}$$

$$F = \frac{K \cdot 3m_1 \cdot m_2}{(3d)^2}$$

$$F = \frac{3K m_1 m_2}{9d^2}$$

$$F = \frac{1}{3} \cdot \frac{K m_1 m_2}{d^2}$$

The force is $\frac{1}{3}$ of what it was

$$\textcircled{11} V = \frac{KT}{P}$$

$$V = \frac{2T}{P}$$

$$20 = \frac{K(300)}{30}$$

$$V = \frac{2(360)}{20}$$

$$600 = 300K$$

$$V = \frac{720}{10}$$

$$2 = K$$

$$V = 36 \text{ ft}^3$$